CONJUGATE PROBLEM OF FREE CONVECTION OF A NON-NEWTONIAN FLUID IN A VERTICAL CHANNEL

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We consider a conjugate problem of free convection of a non-Newtonian fluid with a power law in a plane vertical channel. We discuss the general formulation of the problem, give an analysis of particular cases, and show the effect of the parameters of the problem on convection.

1. There has recently been increased interest in investigations on the convection of nonlinear viscous fluids in channels [1-3]. The problem, however, has still not yet been analyzed in a rigorous formulation.

We consider one-dimensional steady-state convection of a non-Newtonian fluid in a vertical channel with heat-conducting walls. In the fully developed one-dimensional regime the initial system of equations has the form [1]

$$\frac{\partial p}{\partial y} = \frac{d}{dx}\left(k\left|\frac{dv}{dx}\right|^{n-1}\frac{dv}{dx}\right) + g\beta\rho\,(T - T_0) = C, \tag{1}$$

$$0 = \lambda\frac{d^2T}{dx^2} + q. \tag{2}$$

We write the heat-conduction equations for walls with interior sources of heat:

$$\lambda_1\frac{d^2T_1}{dx^2} + q_1 = 0,\ -(h+l) \leqslant x \leqslant -l, \tag{3}$$

$$\lambda_1\frac{d^2T_1}{dx^2} + q_2 = 0,\ l \leqslant x \leqslant (h+l). \tag{3'}$$

In the general case the index of consistency of the fluid k and the parameter of rheological nonlinearity n depend on the temperature. The non-Newtonian properties are weakened during heating, i.e., the thermal dependence of the properties can be determined by the relations ($k \to \mu$, $n \to 1$)

$$k(T) = A\exp(-bT/T_0),\ n(T) = 1 - f(T)\ \text{for}\ \frac{df(T)}{dT} < 0.$$

Then $k \to A\exp(-b) = \mu$, $f(T) \to 0$, and $T \to T_0$, where A and b are constants.

The interior heat source q can be positive or negative, constant or variable.

At the boundaries of the channel with the fluid, the velocity vanishes (condition of adhesion) or is nonzero for the case of reactions (physical and chamical) with the formation of mass inhomogeneities. In addition, the conditions of continuity of temperature and of the normal component of the heat flux should be satisfied. For the temperature on the exterior surfaces we can be given boundary conditions of the first or second kind.

2. We consider the case of temperature-independent thermophysical properties. For distance, velocity, pressure, and temperatue, we choose, respectively, the four scales l, $V = (\rho g\beta\Delta T l^{n+1}/k)^{1/n}$; $P = \rho g\beta\Delta T l$, ΔT, where $2\Delta T$ is the temperature difference between the walls. By introducing the dimensionless quantities $\tilde{x} = x/l$, $\Theta = (T - T_0)/\Delta T$, $\tilde{v} = v/V$, and $\tilde{p} = p/P$, we transform Eqs. (1)-(3') to the form (the ∿ symbol is dropped)

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$$\Theta+\frac{d}{dx}\left(\left|\frac{dv}{dx}\right|^{n-1}\frac{dv}{dx}\right)=\frac{dp}{dy}=F, \tag{4}$$

$$\frac{d^2\Theta}{dx^2}+s=0, \tag{5}$$

$$\frac{d^2\Theta_1}{dx^2}+s_1=0,\ -\left(1+\frac{h}{l}\right)\leqslant x\leqslant 1, \tag{6}$$

$$\frac{d^2\Theta_1}{dx^2}+s_2=0,\ 1\leqslant x\leqslant\left(1+\frac{h}{l}\right), \tag{7}$$

$$x=\pm 1:\ v=0;\ \Theta=\Theta_1,\ \frac{d\Theta}{dx}=\frac{\lambda_1}{\lambda}\frac{d\Theta_1}{dx},$$

$$x=\pm\left(1+\frac{h}{l}\right):\quad \Theta_1=\pm 1,\ \int\limits_{-1}^{+1}v(x)\,dx=0. \tag{8}$$

We represent the boundary conditions for the temperature of the fluid in a different form. Single integration of Eqs. (6) and (7) with the appropriate boundary conditions (8) leads to the equations

$$x=-1:\ \frac{d\Theta}{dx}=\frac{1+\Theta}{\psi}-\frac{s}{2\psi}\left(\frac{h}{l}\right)^2;$$

$$x=+1:\ \frac{d\Theta}{dx}=\frac{1-\Theta}{\psi}+\frac{s_2}{2\psi}\left(\frac{h}{l}\right)^2, \tag{9}$$

where the coupling parameter $\psi = \lambda h/\lambda_1 l$ characterizes the ratio of the thermal conductivity and the thickness in the liquid-wall system. For $x=\pm\left(1+\frac{h}{l}\right)$ in agreement with [4] we have $\Theta(\pm 1)=\pm 1$.

We write the solution of Eq. (5) for the temperature field with the boundary conditions (9)

$$\Theta(x)=\frac{s}{2}(1+2\psi)+\left(\frac{h}{2}\right)^2\frac{(s_2+s_1)}{4}+\frac{4+\left(\frac{h}{l}\right)^2(s_2-s_1)}{4}x-\frac{s}{2}x^2. \tag{10}$$

Substituting (10) into (4) we obtain

$$\frac{d}{dx}\left(\left|\frac{dv}{dx}\right|^{n-1}\frac{dv}{dx}\right)=F-E-2Bx+3Dx^2. \tag{11}$$

Integrating (11), we obtain the velocity distribution in the fluid

$$v(x)=\int\left|Dx^3-Bx^2+(F-E)x+D_1\right|^{1/n}\operatorname{sign}(Dx^3-Bx^2+(F-E)x+D_1)\,dx+D_2. \tag{12}$$

Here, we use the notation

$$3D=\frac{s}{2};\ 2B=\frac{4+\left(\frac{h}{l}\right)^2(s_2-s_1)}{4(1+\psi)},$$

$$E=\left(\frac{h}{l}\right)^2\frac{(s_2+s_1)}{4}+\frac{s}{2}(1+2\psi);$$

and D_1 and D_2 are integration constants, determined from the boundary conditions for the velocity.

3. We consider particular cases. For a Newtonian fluid (n = 1), the velocity profile has the form

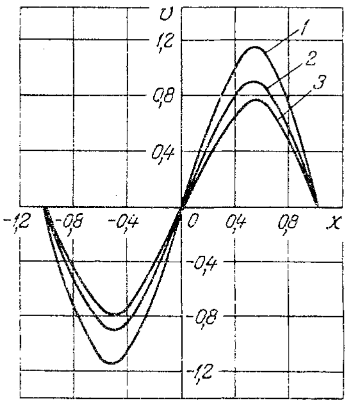

Fig. 1. Velocity profiles for $\psi = 3$ for various values of n: 1) $n = 2$, 2) $n = 3$, $n = 4$.

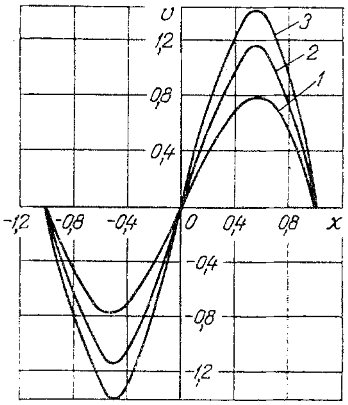

Fig. 2. Velocity profiles for $n = 2$ for various values of ψ: 1) $\psi = 1$, 2) $\psi = 2$, 3) $\psi = 3$.

$$v(x) = \frac{s}{24}(x^4 - 1) + \frac{4 + \left(\frac{h}{l}\right)^2 (s_2 - s_1)}{24(1+\psi)}(x - x^3) - \frac{s}{20}(x^2 - 1).$$

Such a velocity distribution was considered in [5]. We note that for heat sources in the walls that are equal in intensity ($q_1 = q_2$), the velocity of the fluid ceases to depend on them.

In the absence of a heat source in the fluid ($q = 0$) and for very thin walls ($h = 0$) or for $\lambda_1 \to \infty$ ($\psi = 0$), the velocity distribution has the form

$$v(x) = \int_0^x \left| -\frac{x^2}{2} + D_1 \right|^{1/n} \operatorname{sign}\left(\frac{x^2}{2} + D_1\right) dx.$$

This result agrees with that obtained earlier [6].

For the case $s = s_1 = s_2 = 0$ the temperature and velocity profiles are described by the relations

$$\Theta(x) = \frac{x}{1+\psi},$$

$$v(x) = \int_0^x \left| -\frac{x^2}{2(1+\psi)} + D_1 \right|^{1/n} \operatorname{sign}\left(-\frac{x^2}{2(1+\psi)} + D_1\right) dx. \quad (13)$$

The integration constant D_1 is determined from the boundary condition

$$\int_0^1 \left| -\frac{x^2}{2(1+\psi)} + D_1 \right|^{1/n} \operatorname{sign}\left(-\frac{x^2}{2(1+\psi)} + D_1\right) dx = 0. \quad (14)$$

The results (13) and (14), calculated on a computer, are shown in Figs. 1 and 2. The velocity profiles have two extrema with equal absolute values. For a fixed value of ψ (Fig. 1) with increasing n for a dilatant fluid ($n > 1$) the velocity distribution acquires narrower extrema, which agrees with the data of [6]. For a single fluid (the value of n is fixed) with increasing coupling parameter ψ the convection is weakened (Fig. 2). Such mechanical behavior agrees with the physical considerations.

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METHOD OF STUDYING VISCOELASTIC PROPERTIES OF POROUS POLYMER FILMS

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A method is proposed for studying the viscoelasticity of polymer films with the aid of a quartz resonator. Results are reported pertaining to thin copper films.

In order to produce reliable quartz-type air humidity transducers, devices of great practical importance, one must know the viscoelastic properties of thin porous coatings. In another study [1] was demonstrated the feasibility of using a compound vibrator for solid specimens of sections smaller than the end section of the piezoelectric bar. With the use of such specimens it becomes possible to study the viscoelastic properties of polymers in which energy losses are high and the acoustic velocity is low. Unlike a conventional compound vibrator, the one in the method to be described here the piezoelectric cell has a specimen (film) deposited on its entire surface.

The diagram in Fig. 1 depicts an X-cut piezoelectric cell. The excitation electrodes are deposited on the $l_0 l_m$ face. The entire surface of this cell is coated uniformly with a polymer film of thickness Δ' by immersion in a preheated 3-5% solution of the polymer in an appropriate solvent and subsequent spinning in a centrifuge. This process yields then uniform films with excellent adhesion to the surface of the piezoelectric cell. When the latter vibrates and the adhesion is strong, the film will perform the same vibrations as the cell surface, i.e., there will be no sliding of the film. Assuming that the piezoelectric cell vibrates longitudinally along the y axis and that its strains ε are equal to strains in the film, one can write the equation of motion for the cell with film in the form

$$(m_1+m_2)\ddot{\varepsilon}+(N_1+N_2)\dot{\varepsilon}+(G_1+G_2)\varepsilon=F. \tag{1}$$

According to the electrochemical analogy, the electric-circuit equation equivalent to Eq. (1) can be written as

$$(L_1+L_2)\ddot{q}+(R_1+R_2)\dot{q}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)q=U. \tag{2}$$

The relation between parameters in Eq. (1) and those in Eq. (2) can be defined in terms of the coefficient β [2]

$$\beta=2L_1/m_1=2(L_1+L_2)/(m_1+m_2). \tag{3}$$

The equivalent electrical parameters L_1 and L_2, R_1 and R_2, C_1 and C_2 are determined through measurement [3]. The reproducibility of results is high. The resonance frequencies f_1 of the piezoelectric cell and f_2 of the specimen with the cell are determined from Eq. (1), namely

$$f_1=[G_1/2\pi^2 m_1]^{1/2}, \tag{4}$$

$$f_2=[(G_1+G_2)/2\pi^2(m_1+m_2)^{1/2}. \tag{5}$$

Expressions (4) and (5) are used for calculating the stiffness parameters G_1 and G_2. The

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